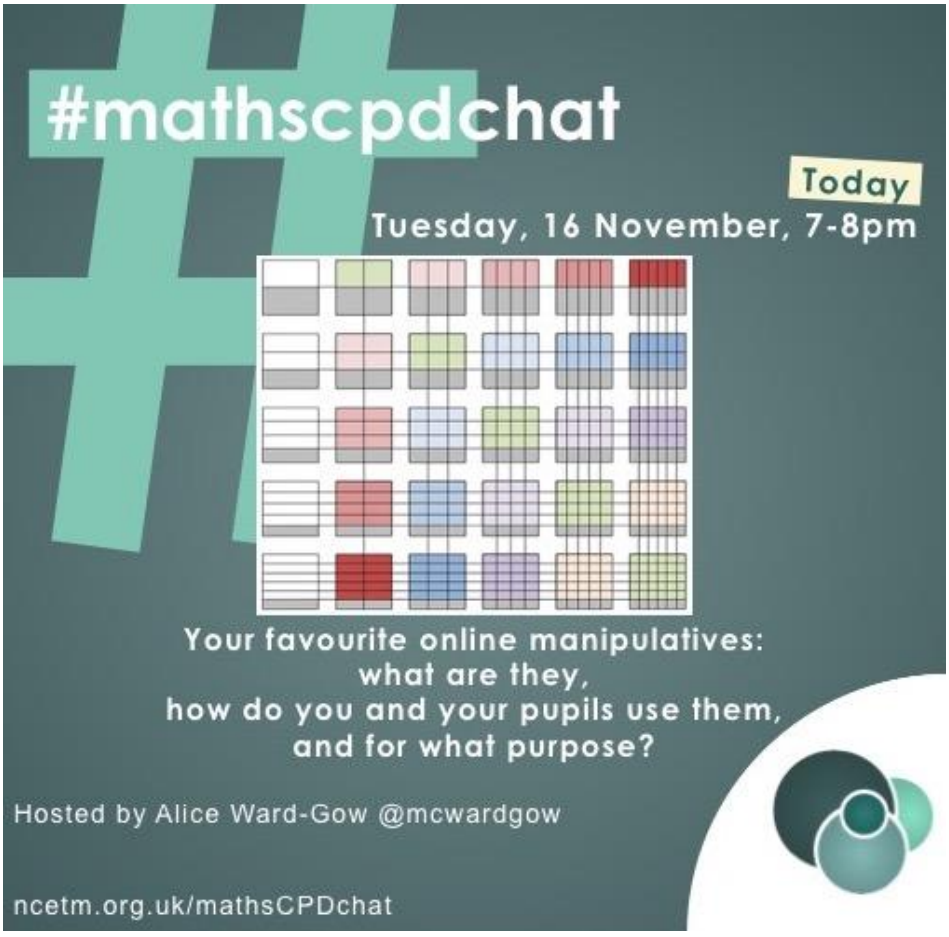
#mathscpdchat 16 November 2021

Your favourite online manipulatives: what are they, how do you and your pupils use them, and for what purpose?
Hosted by Alice Ward-Gow

This is a brief summary of the discussion – to see all the tweets, follow the hashtag ***#mathscpdchat*** *in Twitter*



The links shared during this discussion were:

Mathsbot Virtual Manipulatives which is a popular collection of virtual manipulatives designed and developed by Jonathan Hall. It was shared by Martyn Yeo

These are direct links to the Mathsbot Virtual Manipulatives that were shared during the chat:
Two-Colour Counters ('double-sided counters'), shared by Charlotte Hawthorne and Sharon Malley;
Place Value Counters, shared by Martyn Yeo;
Dienes Blocks, shared by Martyn Yeo, Charlotte Hawthorne and Miss McArdle;
Prime Factor Tiles, shared by Charlotte Hawthorne;
Cuisenaire® Rods, shared by Miss McArdle;
Algebra Tiles, shared by Atul Rana;
Rekenrek, shared by Amy How;
Hundred Square, shared by Mary Pardoe

Polypad which are Mathigon virtual manipulatives, tools and axes. The 'Geometry' manipulatives include Polyominoes, Penrose Tiles, 3-D Solids, Tangram, Pentagon Tilings, and three other kinds of manipulative. The 'Number' manipulatives include 'Number Grids', 'Number Cards', four other kinds of manipulative, and 'Additional Tools'. There are also 'Fraction Bars', 'Fraction Circles', 'Algebra Tiles', a 'Balance Scale', 'Probability and Data' manipulatives, and other tools and axes. It was shared by Barry Smith

Toy Theater Virtual Manipulatives which has a wide variety of many manipulatives, including some less-usual ones, such as 'Marble Jar', 'Beaded Number Line', 'Bear Counters', 'Roman Numeral Tiles'. These manipulatives are particularly suitable for work with very young children. It was shared by Barry Smith

The Math Learning Center: Math Apps which includes a variety of apps, such as 'Geoboard' in which users can stretch virtual bands around pegs to form polygons and represent line segments, and a very adaptable 'number line'. It was shared by Barry Smith

Didax Virtual Manipulatives which is another varied collection of many manipulatives, such as a 'Math Balance', 'Unifix Cubes', 'Geoboard', 'Dice', '120 Number Board', and so on. It was shared by Barry Smith

Visnos - Visual Numbers which are interactive teaching/learning manipulatives/aids, such as an interactive protractor, clocks, spinners, a 'Fractal Explorer', a 'Sieve of Eratosthenes', and an app designed to 'Explore Pi'. It was shared by Barry Smith

SolveMe Mobiles which are interactive balancing tasks in which users 'play with' virtual objects placed on a virtual balance, and reason to solve problems. It was shared by Atul Rana

Equality Explorer which is a virtual manipulative balance on which users place/balance objects and numbers, reason and solve problems. It was shared by Atul Rana

Distributive Law which is an illustrated 'Starting Points Maths' blog by Chris McGrane, in which he shows how algebra tiles may be used to represent, and shed light on, the factorising of, and expanding brackets in, algebraic expressions ('doing and undoing'). It was shared by Alice Ward-Gow

Directed (Negative) Numbers which are tasks from PixiMaths designed to support learning to understand and use (operate with and on) directed numbers. It was shared by Alice Ward-Gow

Cuisenaire® rod equations which are Don Steward tasks in which learners use (real) Cuisenaire® rods to find and explore relationships between numbers. It was shared by Peter Mattock

Videos by Peter Mattock which are YouTube videos in which Peter Mattock demonstrates how various manipulatives may be used to support learners' understanding of, and ability to work with, many different mathematical ideas and procedures. It was shared by Lee Overy

NRICH: Primary Interactive Resources which are general interactive resources, such as geoboards and an Interactive Balance, that can be used in a variety of contexts, and many specific interactive tasks and games, such as 'Light the Lights' or 'Junior Frogs'. It was shared by Mary Pardoe

NRICH: Secondary Interactive Resources which are general interactive resources, such as Interactive Spinners and geoboards, that can be used in a variety of contexts, and many specific interactive tasks and games, such as 'Number Sandwiches' or 'Frogs'. It was shared by Mary Pardoe

Sketch CPD Resources which are attractively illustrated mathematical tasks created/designed by Charlotte Hawthorne, many of which involve/depend-on the use of manipulatives such as algebra tiles or balances. It was shared by Jonathan Hall

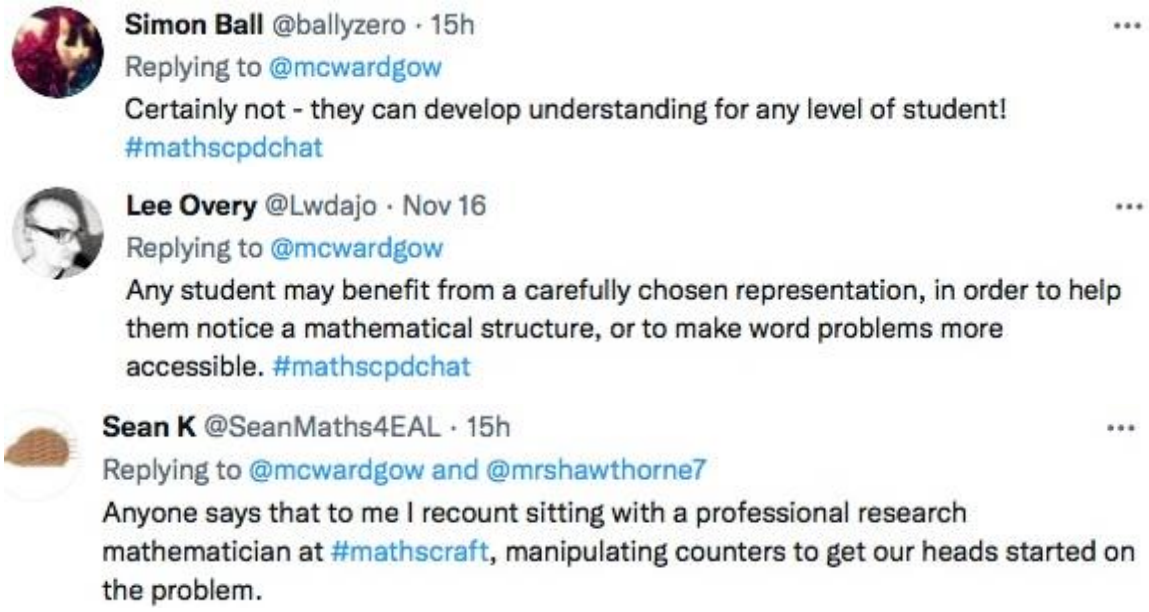
MathsChatLive which is the recording on Twitter of an online Zoom-discussion between maths educators about 'learning how to use manipulatives'. It was shared by Atul Rana

The screenshots below, of chains of tweets posted during the chat, show conversations prompted by Alice's third question, which included a poll. These replies support the poll results, and include justifications, with examples, of teachers' findings that the use of manipulatives can

support school maths learning at any level. **Click on any of these screenshots-of-a-tweet to go to that actual tweet on Twitter.**

The conversations were generated by this question from Alice Ward-Gow:

and included these from Simon Ball, Lee Overy and Sean K:



these from Charlotte Hawthorne, Alice Ward-Gow and Kimberly Crane:

Miss Ward-Gow @mcwardgow · Nov 16

Replying to @mrshawthorne7

Absolutely! If they haven't seen something that makes them go "wow" with manipulatives, maybe if they see that it will give them a different perspective 😄 #mathscpdchat

+ × kimberly crane — ÷ @kimberw5226 · 8h

Replying to @mcwardgow and @mrshawthorne7

I would ask a group of adults with college degrees to model division using an area model built with base 10 blocks. While not beginning learners, I'm quite certain their thinking about that concept will be challenged.

these from justlearn, Stephen Blinkhorn and RHMaths

justlearn @justlearnmaths · 2h

Replying to @mcwardgow

Manipulatives are the first step in moving from the concrete to the abstract, providing a visual that students can always refer to. They always hold a place in the CPA approach from counting bears in EY to algebra tiles for completing the square in y11.

Stephen Blinkhorn @MrBlinkhorn · 15h

Replying to @mcwardgow

I remember seeing algebra tiles for completing the square, blew my mind how easy it made it to comprehend.

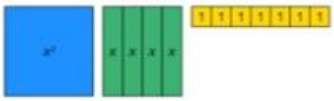
RHMaths @MathsRh · 11h

Replying to @MrBlinkhorn and @mcwardgow

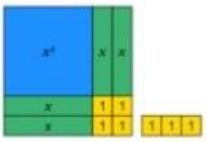
If you haven't already seen 😍 #mathscpdchat

> **Tayyub Majeed** @tm_maths · Oct 27
>
> Visualising completing the square, part of my no-nonsense booklets.

Consider the quadratic expression x^2+4x+7. Visually, this can be demonstrated using algebra tiles:



The idea is to create a rectangular area from these tiles (which in this case is impossible), but to attempt to get as close as possible to a perfect square:



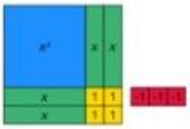
Here, there is a complete square, with a side length of $(x+2)$ units, with three extra 1 tiles.

Hence, the result of completing the square is $x^2+4x+7\equiv(x+2)^2+3$.

Consider the quadratic expression x^2+4x+1. Visually, this can be demonstrated using algebra tiles:

The idea is to create a rectangular area from these tiles (which in this case is impossible), but to attempt to get as close as possible to a perfect square:

Here, we are three 1 tiles short of being able to form a complete square. This can be overcome by adding three 1 tiles with three -1 tiles to form a zero-pair and maintain equivalence.

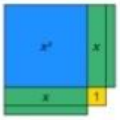


Here, there is a complete square, with a side length of $(x+2)$ units, with three extra -1 tiles.

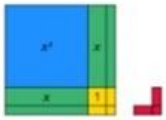
Hence, the result of completing the square is $x^2+4x+1\equiv(x+2)^2-3$.

Consider the quadratic expression x^2+3x+1. Visually, this can be demonstrated using algebra tiles:

The idea is to create a rectangular area from these tiles (which in this case is impossible), but to attempt to get as close as possible to a perfect square. There is a dilemma when there is an odd number of x tiles; we can attach one to the side and one to the bottom, but what about the third? The correct approach is to split the third in half:

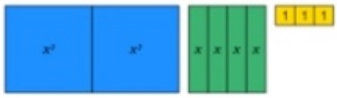


Following the usual procedure of making use of zero-pairs, the completed square can be visualised:



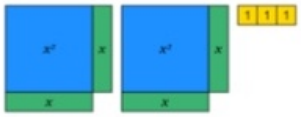
Hence, the result of completing the square is $x^2+3x+1\equiv\left(x+1\frac{1}{2}\right)^2-1\frac{1}{4}$.

Consider the quadratic expression $2x^2+4x+3$. Visually, this can be demonstrated using algebra tiles:

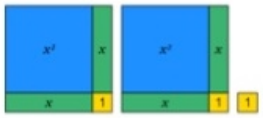


Attempting to get close as possible to a complete square using these tiles is difficult, so the instruction will now be modified to attempting to get as close as possible to completing two squares.

This would be equivalent to rewriting $2x^2+4x+3$ as $2(x^2+2x)+3$, as there is not a need of splitting the 1 tiles.



Instead, you can fill the empty spaces with two 1 tiles, with an extra 1 tile remaining.



Hence, the result of completing the square is $2x^2+4x+3 \equiv 2(x+1)^2+1$.

these from Sharon Malley, Martyn Yeo and Darren Elgar

Sharon Malley @mathsmumof2 · 15h

Replying to @mcwardgow

I had numicon out with top set year 9 to show that even numbers are 2n and odd numbers are 2n-1 and we can then make conjectures about the sum of two numbers and prove visually and algebraically. #mathscpdchat

Martyn (He/Him) @martynyeouk · 15h

Replying to @mcwardgow

Manipulatives are for everyone - in fact on a course I did I was asked to demonstrate a process using manipulatives which was more challenging than using numbers #mathscpdchat

Darren Elgar @ElgarDarren · 14h

Replying to @mcwardgow

I am doing a bit of studying myself, something different to my normal primary.

I was learning about finding the difference between two squares and I modelled it with cuisenaire and algebra tiles. I found it helped me no end to visualise and understand what was going on.

these from Alice Ward-Gow , Mary Pardoe and Jonathan Hall:

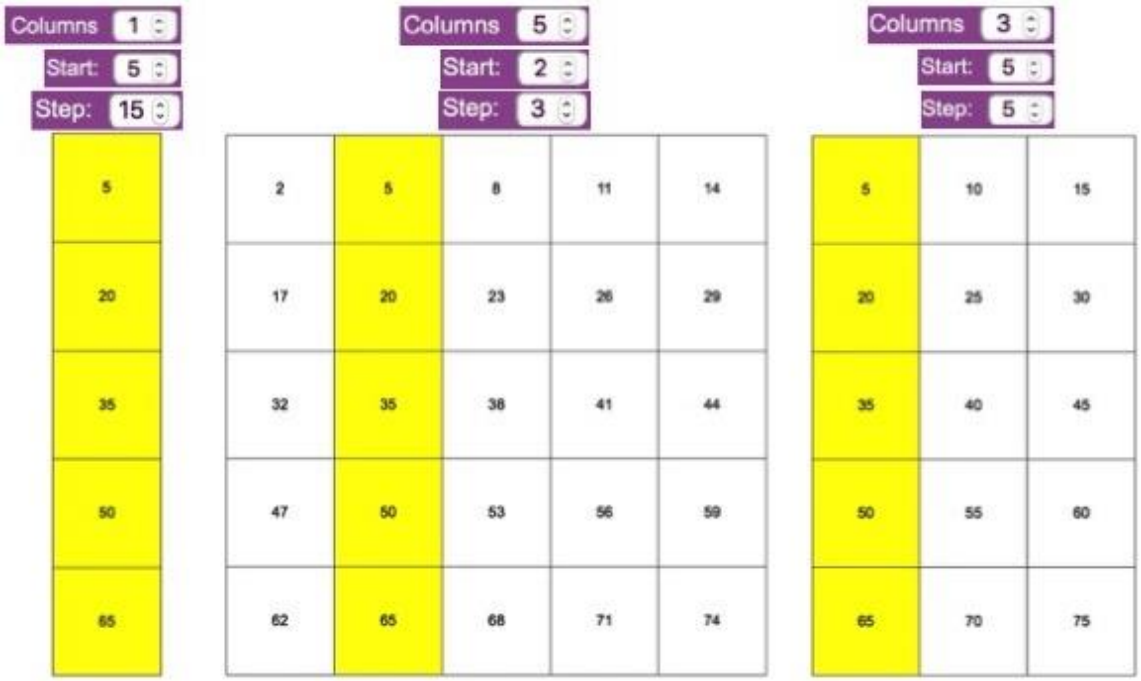
Miss Ward-Gow @mcwardgow · Nov 16

Q3. A colleague says "manipulatives are for lower ability students." Do you agree with this comment? Explain why? **#mathscpdchat**

Mary Pardoe @PardoeMary · 15h

Replying to @mcwardgow

Definitely disagree! The other day I was playing with @StudyMaths 's online hundred square. I set it up at random, then asked myself 'how else could I get THAT column?' Not that easy! Eg ... more possibilities ... ? #mathsCPDchat
That online manipulative: mathsbot.com/manipulatives/...



Mary Pardoe @PardoeMary · 15h

... another one I started to look for ...
#mathsCPDchat
mathsbot.com/manipulatives/...

Columns: 1 Start: 4 Step: 12

4
16
28
40
52
64
76
88
100

Columns: 4 Start: 1 Step: 3

1	4	7	10
13	16	19	22
25	28	31	34
37	40	43	46
49	52	55	58
61	64	67	70
73	76	79	82
85	88	91	94
97	100	103	106

Columns: 6 Start: 2 Step: 2

2	4	6	8	10	12
14	16	18	20	22	24
26	28	30	32	34	36
38	40	42	44	46	48
50	52	54	56	58	60
62	64	66	68	70	72
74	76	78	80	82	84
86	88	90	92	94	96
98	100	102	104	106	108

Jonathan Hall @StudyMaths · 15h

Nice! I love seeing what people get up to with my site.
#mathscpdchat

these from CantabKitty and Alice Ward-Gow:

CantabKitty BSc @CantabKitty · Nov 16

Replying to @mcwardgow and @mrshawthorne7

The reason I don't use them with my higher sets is that there are 15 packs of algebra tiles in a pack and our higher sets have 33 students in them.

Miss Ward-Gow @mcwardgow · Nov 16

But if you had more, you would use them with higher sets? #mathscpdchat

CantabKitty BSc @CantabKitty · Nov 16

Yep.

CantabKitty BSc @CantabKitty · 15h

I think my high set y7s did well even if we did spend 5 minutes moving chairs so they could work in 3s

and these from Miss McArdle, Alice Ward-Gow, Amy How and Joelle:

Miss McArdle @McArdleNumeracy · 15h
Replying to @mcwardgow
I think it is sometimes more obvious how to use manipulatives for simpler concepts. So as a teacher the way a topic is introduced and questions/prompts used have a big impact on how effective the use of manipulatives is for that learning experience.

#MathsCpdChat

Miss Ward-Gow @mcwardgow · 15h
You're absolutely right - that introduction is key isn't it. How long would you spend on an introduction? 🙂 #mathscpdchat

Miss McArdle @McArdleNumeracy · 15h
I don't have a set length of time for intro, but I start with a question that is easier with the manipulative, e.g.

Which is larger - 1 fifth or 1 eighth? Many may already know this.

Which is larger - 3 fifths or 5 eighths? This is so quick to prove with tiles.

#MathsCPDChat

Miss Ward-Gow @mcwardgow · 15h
Really like that example - building on students prior knowledge 🙂 thank you for sharing #mathscpdchat

Amy How @rekenrek101 · 15h
Replying to @mcwardgow
I can honestly say children using manipulatives are able to reason - explain , describe, justify and prove their understanding much easier. #mathscpdchat

Joelle @MinsterMaths · 5h
Replying to @mcwardgow
The manipulative allow students to fully understand the structures of the problem. Too often this step is omitted for high ability and many students become algorithmic learners as a result. Careful use allows investigation and discovery in my opinion.

(to read the discussion sequence generated by any tweet, look at the 'replies' to that tweet)

The host's third question generated more replies and conversations than any other question, and screenshots of most of those replies are shown above. Replies to her first question …

Miss Ward-Gow @mcwardgow · 15h

Welcome to tonight's #mathscpdchat on manipulatives - don't forget the hashtag!

Q1. You're told that you'll be taking part in a CPD session on manipulatives. What's your initial reaction - GIFs only!

… included:

Sharon Malley @mathsmumof2 · 15h
Replying to @mcwardgow
#mathsCPDchat

Kirsty Fish @Kirstymaths · 14h
Replying to @mcwardgow
#mathscpdchat

Charlotte Hawthorne @mrshawthorne7 · 15h
Replying to @mcwardgow

Alice's second and last questions …

… together prompted many suggestions of resources that teachers have used successfully, and links to them are provided above. Some of those suggestions generated conversations, such as this one, in which a more general difference in typical ways of working in KS2 and KS3 was mentioned:

Miss Ward-Gow @mcwardgow · 15h

I saw the same when I went to a primary school pre-covid. The students were able to choose which manipulatives they wanted to use to solve a problem. Why do you think this changes at secondary? #mathscpdchat

Sharon Malley @mathsmumof2 · 15h

Students arriving into the space and therefore sharing it with other classes rather than one classroom and 'their stuff' #mathscpdchat

In the following reply, a reason was given for using virtual, rather than real, manipulatives:

Charlotte Hawthorne @mrshawthorne7 · 15h

Replying to @mcwardgow

I'm struggling to choose! I love dienes for being able to use different bases and going beyond 1000 on MathsBot.com things I can't do with real dienes. Love directed counters where the size of the counters change. Love prime factor tiles. #mathsCPDchat

And a difference in the nature of experiences when using virtual compared with real manipulatives was also mentioned in this tweet:

Miss McArdle @McArdleNumeracy · 14h

Replying to @mcwardgow

I love that (if each child has own device) no sharing and there's no tidying up, but there's nothing quite the same as having the physical resource in your hand for some topics.

Online and real life, I probably use Dienes, Cuisenaire and fraction tiles the most.

#MathsCPDChat

Alice's fourth question …

Miss Ward-Gow @mcwardgow · 16h

Q4. Which topics lend themselves nicely to manipulatives? 😄 #mathscpdchat

… provided an opportunity to think about the use of virtual balance scales to represent/support learning about solving linear equations:

Lee Overy @Lwdajo · 16h

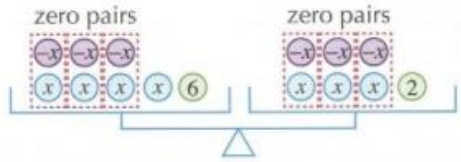
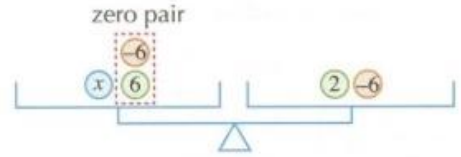
Replying to @mcwardgow

Algebra discs can be used as a common representation with operations with number and algebraic manipulation (from simplifying expressions through to factorising quadratics). A familiar representation has the advantage of students being "used to it". #mathscpdchat

Lee Overy @Lwdajo · 16h

Since @studymaths is here, it would be nice to see a balance scales option on the algebra discs page in order to solve linear equations (unless I missed it)! #mathscpdchat

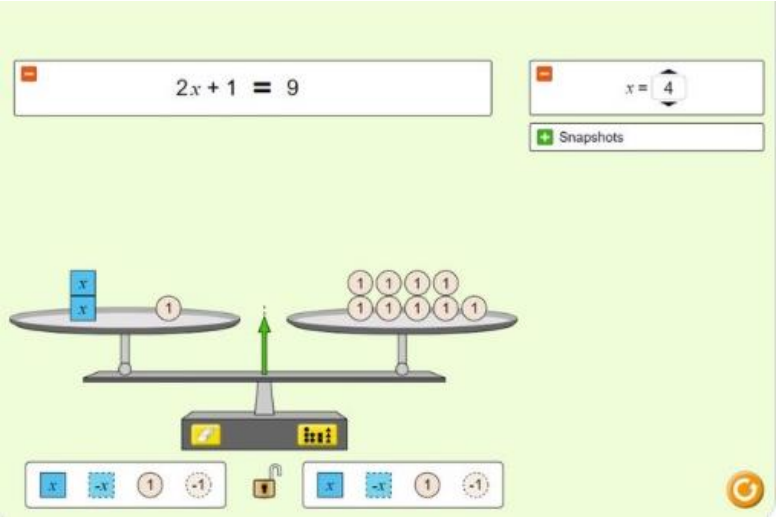
Example: $2(2x + 3) = 3x + 2$

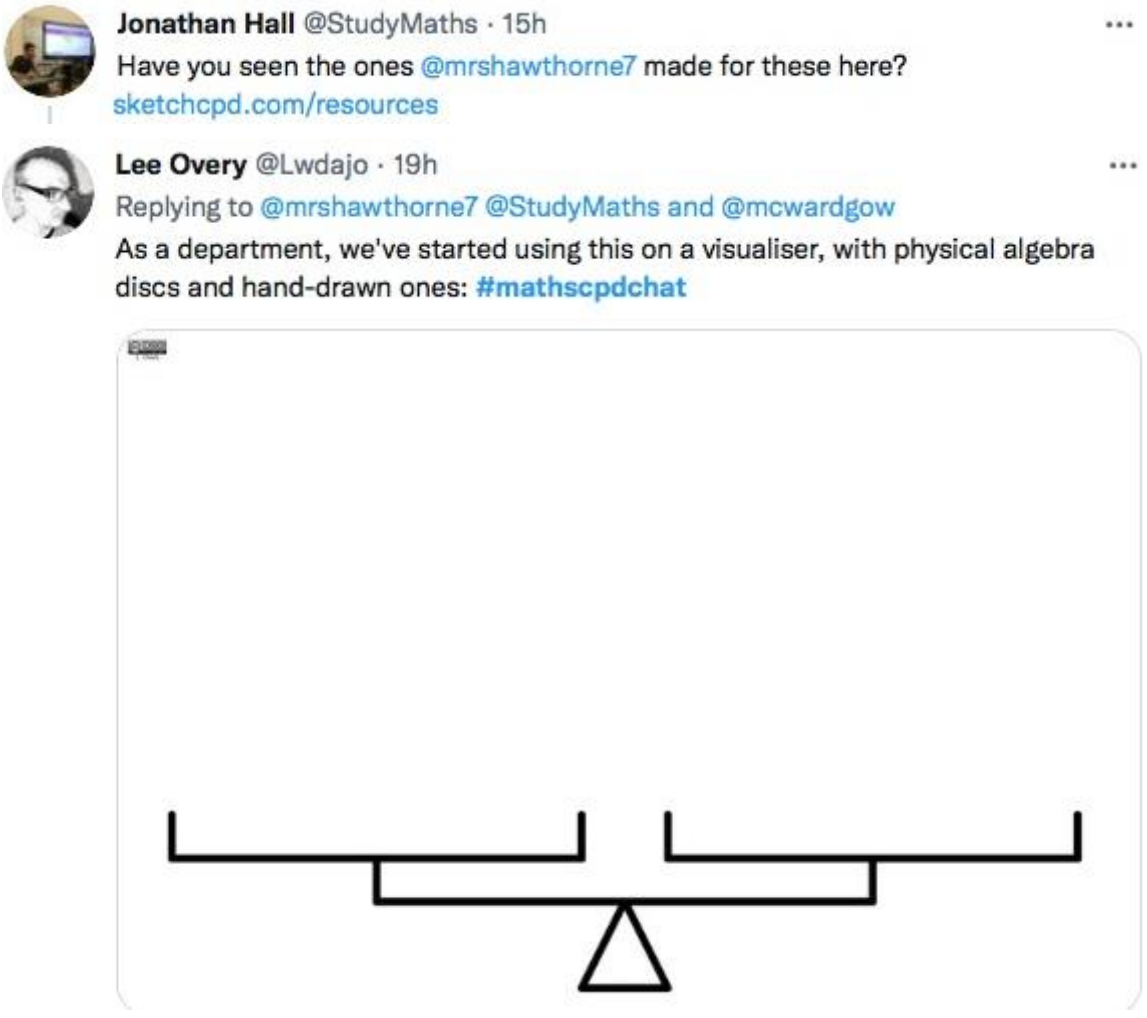
	$2(2x + 3) = 3x + 2$
	Expand the expression on the LHS: $4x + 6 = 3x + 2$
Add 3 $-x$ to both scale pans. zero pairs zero pairs 	Add $-3x$ to both sides: $4x - 3x + 6 = 3x - 3x + 2$
	Simplify both sides: $x + 6 = 2$
Add -6 to both scale pans. zero pair 	Add -6 to both sides: $x + 6 - 6 = 2 - 6$
	Simplify both sides: $x = -4$

Atul Rana @atulrana · 3h

Replying to @Lwdajo @mcwardgow and @StudyMaths

Miss this last night...here are the virtual manipualtives I use for equality and equations #MathsCPDchat





Also mentioned in response to the fourth question were:

- **using Geogebra or Desmos when working with graphs,**
- **spreadsheets for statistics,**
- **Cuisenaire® rods for learning about multiplication and fractions.**